BT–MRE–I-12.19-1471 Reg. No. [| | | | | | | |]

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B.Tech. Degree I Semester Examination in Marine Engineering December 2019

19-208-0101 ENGINEERING MATHEMATICS I

(2019 Scheme)

Time : 3 Hours Maximum Marks : 60

(5 × 15 = 75)

I. (a) Find the equation of the asymptote of the hyperbola $x^2 + xy - 2y^2 + 5x + y - 6 = 0$. (5)

(b) Find the eccentricity, foci, directrices and the length of the latus rectum of the ellipse $2x^2 + 3y^2 = 6$. (5)

(c) Find the equation of the parabola which is symmetric about the Y-axis and passes through the point (2,-3). (5)

OR

II. (a) Show that $lx + my + n = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $n^2 = a^2l^2 - b^2m^2$. (5)

(b) Show that the locus of point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is the directrix $x + a = 0$. (5)

(c) Obtain the equation of an ellipse whose focus is (3,1) and directrix is $x - y + 6 = 0$ and eccentricity is $\frac{1}{2}$. (5)

III. (a) Verify Rolle's theorem for the function $f(x) = log\left\{\frac{x^2 + ab}{x(a+b)}\right\}$ on [a,b] where 0<a<b. (5)

(b) Using reduction formula integrate (5)

(i) $\int sin^4 x cos^2 x \, dx$

(ii) Evaluate $\int_0^{\frac{\pi}{2}} sin^3 x cos^7 x$ dx.

(c) Find the volume of the reel formed by the revolution of the cycloid $x = a(\theta + sin\,\theta)$, $y = a(1 - cos\theta)$ about the tangent at the vertex. (5)

OR

IV. (a) Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{sec^2 x - 2\tan x}{1 + \cos 4x}$. (5)

(b) Show that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$ is $4a\cos\frac{\theta}{2}$. (5)

(c) Find the surface area of the solid formed by revolving the cardioids $r = a(1 + cos\theta)$ about the initial line. (5)

(P.T.O.)

V (a) Find the n^{th} derivative of $\dfrac{10x-21}{(2x-3)(2x+5)}$. (5)

(b) If $y=\cos(m\sin^{(-1)} x)$, prove that $\left(1-x^2\right)y_{n+2}-(2n+1)x\,y_{n+1}+\left(m^2-n^2\right)y_n=0$. (5)

(c) If u is a homogeneous function of degree 'n' in x and y, then prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=nu$. (5)

OR

VI. (a) Find the maxima and minima of the function $f(x,y)=x^3+y^3-3x-12y+20$. (5)

(b) Verify Euler's theorem for the function $f(x,y)=\dfrac{x(x^3-y^3)}{x^3+y^3}$. (5)

(c) If $r^2=(x-a)^2+(y-b)^2+(z-c)^2$, then prove that $\dfrac{\partial^2 r}{\partial x^2}+\dfrac{\partial^2 r}{\partial y^2}+\dfrac{\partial^2 r}{\partial z^2}=\dfrac{2}{r}$. (5)

VII. (a) Find by double integration, the area lying inside the circle $r=a\,sin\theta$ and outside the cardioid $r=a\left(1-cos\theta\right)$. (5)

(b) Evaluate $\displaystyle\int_0^1\int_0^1\int_{z=\sqrt{x^2+y^2}}^{2} xyz\,dz\,dy\,dx$. (5)

(c) Obtain a set of vectors reciprocal to the vectors, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}+2\hat{k}$. (5)

OR

VIII. (a) Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-\hat{k}$. (5)

(b) Prove that $\left[\vec{a}+\vec{b}\;\;\vec{b}+\vec{c}\;\;\vec{c}+\vec{a}\right]=2\left[\vec{a}\;\;\vec{b}\;\;\vec{c}\right]$. (5)

(c) If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$, then verify that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$. (5)

IX. (a) Verify Stokes theorem for $\vec{F}=\left(x^2+y^2\right)\hat{i}-2xy\hat{j}$ taken round the rectangle bounded by the lines $x=\pm a$, $y=0$, $y=b$. (5)

(b) Show that $\vec{f}=\left(y^2\cos x+z^3\right)\hat{i}+\left(2y\,sin\,x-4\right)\hat{j}+\left(3xz^2\right)\hat{k}$ is a conservative field. (5)

(c) Find the directional derivative of $\varphi(x,y,z)=xy^2+yz^3$ at the point (2,-1,1) in the direction of the vector $4\hat{i}+2\hat{j}+4\hat{k}$. (5)

OR

X. (a) Find $div\,\vec{F}$ and $curl\,\vec{F}$ where $\vec{F}=grad\left(x^3+y^3+z^3-3xyz\right)$. (5)

(b) A vector field is given by $\vec{A}=\left(x^2+xy^2\right)\hat{i}+\left(y^2+x^2y\right)\hat{j}$, show that the field is irrotational and find the scalar potential. (5)

(c) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, -1, 2). (5)

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